Proceedings of:
Current Research in Hydropower Technologies, CRHT X
Kathmandu University, 2020
CRHTX-03

Average-Value Modelling of Machine-Converter Systems for Variable Speed Hydropower

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Abstract. The development in frequency converter technology has enabled the use of converter-fed synchronous machines for variable speed operation of hydropower plants. This is beneficial due to improvements in hydraulic efficiency and increased flexibility for the power grid. Because system studies involving such machine-converter based system can be quite computational expensive, there is a need for more simplified models. Average-value models are effective tool for simplifying machine-converter systems and may execute simulations magnitudes faster than detailed models. This paper will present the work done in the preliminary work for the author's master thesis. The focus of this work was to implement an average-value model for a machine-rectifier system and to assess the accuracy of the developed model. In the successive master thesis, this work will be expanded to a variable speed hydropower plant, consisting of a full-rated converter-fed synchronous machine.

Keyword: Variable Speed Hydropower, System Studies, Computer Modelling, Average-Value Modelling.

1. Introduction

By use of full-rated frequency converter solutions for hydropower plants, the frequency/speed of the electrical machine and turbine is decoupled from the frequency of the grid thus enabling variable speed operation of the hydropower plant [1]. Variable speed operation of hydropower plants can be advantageous for both the hydraulic system and the power system. Hydropower plants offer flexible production to the power grid which is important for balancing the grid during power oscillations. However, this has led to the hydropower plants being operated at off-design conditions, creating hydraulic phenomena that reduces the operating life of the plant. Furthermore, the use of frequency converter interfaced hydropower plants will minimize the negative influence of the power plant on the system by being able to control power factor at the grid side. Increasing the flexibility of hydropower plants will also be important as more intermittent generation such as wind and solar is introduced in the power system [2].

The modelling of variable speed hydropower plants can be problematic because of the differences between the dynamics of the hydraulic and the electric system; The hydraulic system is characterised by long time-constant and need time simulation over a long time period, whereas frequency converters with high switching frequencies will need to be simulated with short time steps. As the detailed simulation of the switching elements of the frequency converters can be quite computational expensive, this means that simulations of variable speed hydropower systems might become excessively computational expensive and time consuming - leading to reduced size and complexity of the modelled systems [3].

However, these systems can be simplified by only considering the most important dynamics of the system in what is called average-value models. In these models, the higher order harmonics of the electrical ac signals are neglected, while dc signals are averaged within a converter switching interval. The use of average-value models allows the time steps to be increased, decreases simulation time and enables the simulation of more complex models.

One method of average-value modelling is the parametric average-value model (PAVM) method, where converter relationships are described through algebraic functions based on dynamic studies on a detailed model of the system [3-4].

As the preliminary work of the author's master thesis, the use of a PAVM for a machine-rectifier system was examined: A detailed model a synchronous machine-fed diode rectifier and dc-load was modelled in Simulink (Sec.2), A parametric average-value model was developed based on the detailed Simulink model (Sec.3), Finally, steady-state and dynamic simulations were conducted in order to assess the accuracy of the developed PAVM (Sec.4). In (Sec.5) and (Sec.6). a short discussion and conclusion of the work is presented.

2. Machine-Rectifier System Description

In figure 1, an illustration of the detailed system can be seen. Machine-fed rectifier systems are extensively used for variable speed applications in for example marine power systems, wind power systems and aircraft power generation. The machine-rectifier model was built using the *Specialized Power Systems* library in Simulink. In this library package, a synchronous machine block, universal bridge block and capacitor was used to create the system. Parameter values were based on commonly used reference values and can be found in appendix A.1 for the synchronous machine, diode rectifier and dc-link. A variable resistive element was used to simulate the varying load on the dc-link, while a standard Simulink excitation system based on IEEE type AC1A was used to maintain constant voltage at the machine terminals. If speed was set as an input variable to the synchronous machine block, the output power could vary freely within the machine's capability. The speed was set to a fixed value since the rectifier average-value model would be valid for all rotational speeds. The operation of this system in variable speed was also in focus in the author's preliminary work of the master thesis; However, the results from those system study simulations where not relevant for this paper.

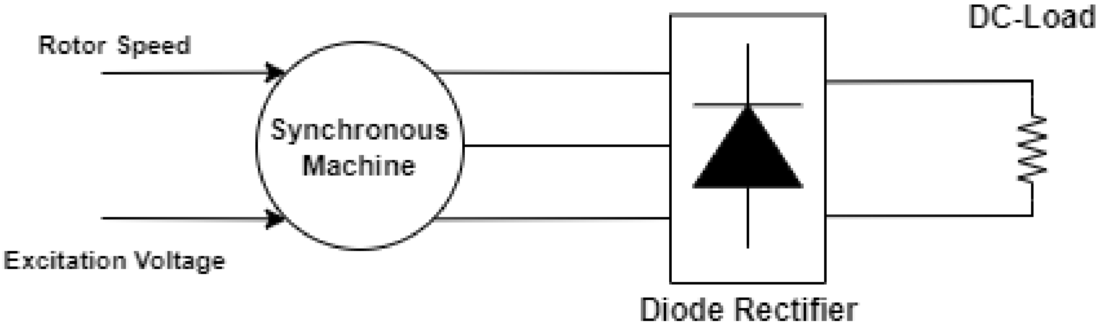

Figure 1. Simplified illustration of the detailed machine-rectifier system created in Simulink.

3. Average-Value Modelling

For an electrical machine delivering large amounts of power through a frequency converter, the voltage and current waveforms will be notably distorted. This problem can be solved by transforming the variables from abc-quantities to qd-axis quantities, as defined in figure B.1a in the appendix. This ensures that the variables are constant dc-quantities in steady-state. The next step is a so-called "fast-average" of the qd-axis reference frame variables within the switching period:

$$\bar{x} = \frac{1}{T}\int_{t-\mathrm{T}}^{t} x(t)\, dt \qquad (1)$$

Furthermore, it is usual to express the qd-axis variables in the rectifier reference frame, as illustrated in figure B.1b [**1**], aligning the q-axis with the a-phase of the rectifier input and setting the d-axis to zero.

Continuing, three algebraic functions need to be established for the diode rectifier: a relation between input voltage and output dc-voltage, a relation between input current and output dc-current and the angle between the input voltage and current. These are defined by equations (2)-(4) below:

$$\left|\overline{V}_{qd}^{rec}\right| = \alpha(z) \times \overline{v}_{DC} \tag{2}$$

$$\overline{\imath}_{DC} = \beta(z) \times \left|\overline{\imath}_{qd}^{rec}\right| \tag{3}$$

$$\phi(z) = tan^{-1}\left(\frac{\overline{\imath}_d}{\overline{\imath}q}\right) - tan^{-1}\left(\frac{\overline{v}_d}{\overline{v}q}\right) \tag{4}$$

Through detailed simulations of the Simulink model, values for α, β and ϕ are defined for a wide range of loading conditions. It is common to define the loading of the dc-link by a dynamic impedance Z as seen in equation 4:

$$z = \frac{\overline{v}_{DC}}{\left|\overline{\imath}_{qd}^{rec}\right|} \tag{5}$$

The load on the dc-link is varied and simulated under steady-state for 14 different loading conditions. The voltage, current and angle relation found from the model is visualized in figure 2, while its support points can be found in the appendix B.2. The data from table B.2 was used to define dynamic look-up tables that was used to implement the parametric average-value model. The rectifier average-value model is implemented with a sixth order state-space representation of the synchronous machine, the same which is used for its implementation in standard Simulink synchronous machine blocks.

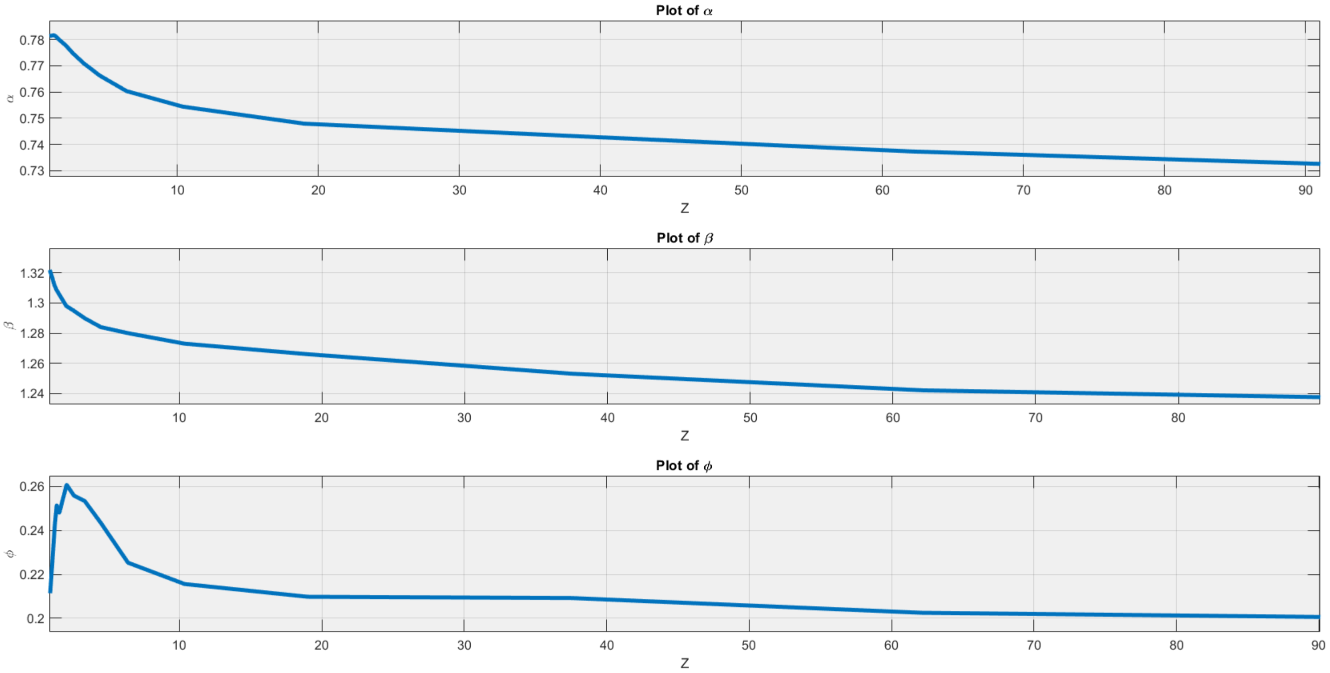

Figure 2. Graphical plot of α, β and ϕ as a function of the dynamic impedance z.

3.1. Verification of the average-value model

The proposed average-value model was assessed for steady-state operation at varying load and for load steps. In this work, the focus was on examining the accuracy of the predicted variables by the rectifier average-value model. The assessed variables were the dc-link current and voltage, load angle of the machine and the qd-axis voltages. For comparison, another type of average-value model, an analytical average-value model (AAVM), was implemented as well. The analytical model was based on the procedure found in [5].

4. Results

4.1. Steady-state simulations

Table 1. Steady-state accuracy of AVMs at partial loads.

Model	$P_{DC} = 0.3$ pu	$P_{DC} = 0.4$ pu	$P_{DC} = 0.5$ pu
Detailed model (reference)	$i_{DC} = 147.4$ A $v_{DC} = 515.6$ V	$i_{DC} = 196.0$ A $v_{DC} = 509.4$ V	$i_{DC} = 251.8$ A $v_{DC} = 503.1$ V
PAVM	$i_{DC} = 147.3$ A *(error = 0.036 %)* $v_{DC} = 515.8$ V *(error = 0.027 %)*	$i_{DC} = 196.1$ A *(error = 0.074 %)* $v_{DC} = 509.7$ V *(error = 0.076 %)*	$i_{DC} = 252.0$ A *(error = 0.081 %)* $v_{DC} = 503.3$ V *(error = 0.035 %)*
AAVM	$i_{DC} = 148.7$ A *(error = 0.794 %)* $v_{DC} = 521.5$ V *(error = 1.100 %)*	$i_{DC} = 198.7$ A *(error = 1.364 %)* $v_{DC} = 516.5$ V *(error = 1.411 %)*	$i_{DC} = 254.6$ A *(error = 1.096 %)* $v_{DC} = 508.7$ V *(error = 1.230 %)*

Table 2. Steady-state accuracy of AVMs at high loads.

Model	$P_{DC} = 0.8$ pu	$P_{DC} = 0.9$ pu	$P_{DC} = 1.0$ pu
Detailed model (reference)	$i_{DC} = 409.1$ A $v_{DC} = 491.0$ V	$i_{DC} = 464.6$ A $v_{DC} = 488.7$ V	$i_{DC} = 513.3$ A $v_{DC} = 486.1$ V
PAVM	$i_{DC} = 408.8$ A *(error = 0.060 %)* $v_{DC} = 490.7$ V *(error = 0.055 %)*	$i_{DC} = 463.3$ A *(error = 0.329 %)* $v_{DC} = 487.4$ V *(error = 0.271 %)*	$i_{DC} = 512.0$ A *(error = 0.263 %)* $v_{DC} = 485.4$ V *(error = 0.149 %)*
AAVM	$i_{DC} = 408.6$ A *(error = 0.099 %)* $v_{DC} = 490.5$ V *(error = 0.102 %)*	$i_{DC} = 460.7$ A *(error = 0.848 %)* $v_{DC} = 482.8$ V *(error = 1.231 %)*	$i_{DC} = 505.7$ A *(error = 1.907 %)* $v_{DC} = 480.1$ V *(error = 1.238 %)*

4.2. Load-step simulations

In figures 3-5, the predicted dc-link current and machine qd-axis voltages can be seen. The presented results are from the simulation of a load step from 0.4 pu (100 kW) to 0.5 pu (125 kW) of the synchronous machines rated power. The dc-link current is the output from the PAVM to the dc-link circuit containing the load, while the qd-axis voltages are the input back to the synchronous machine state-space model. Two other load step scenarios were also simulated, but due to similarity the below figures and space constraints in this paper, they are not presented.

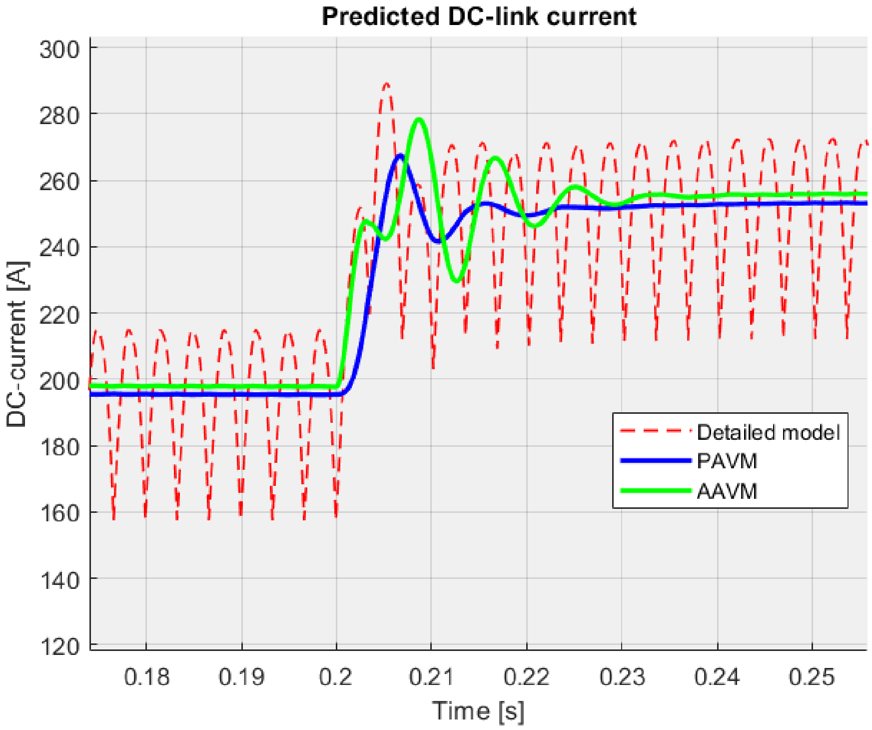

Figure 3. Predicted dc-link current by the parametric average-value model (PAVM). Compared with an analytical average-value model (AAVM) and the detailed model.

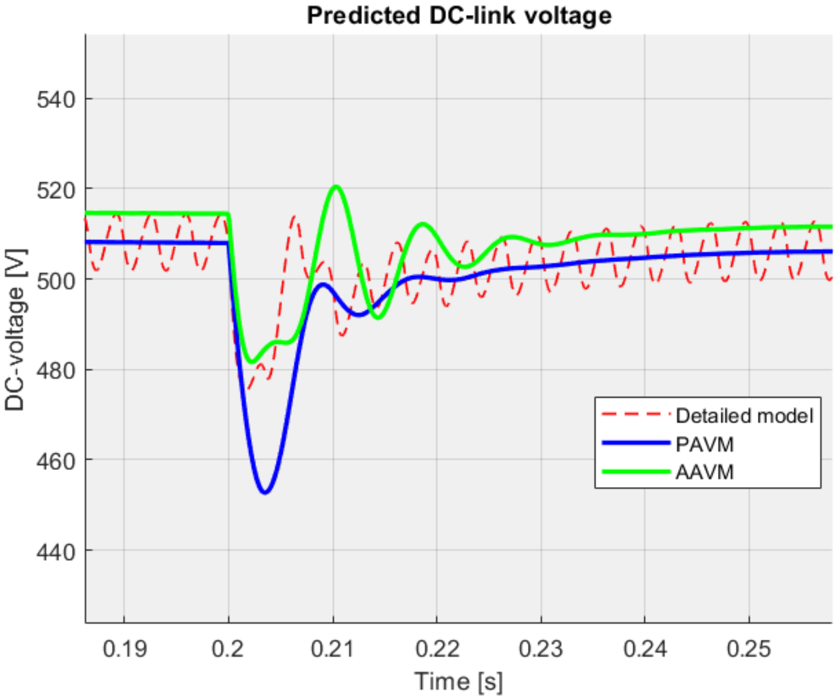

Figure 4. Predicted dc-link current by the parametric average-value model (PAVM). Compared with an analytical average-value model (AAVM) and the detailed model.

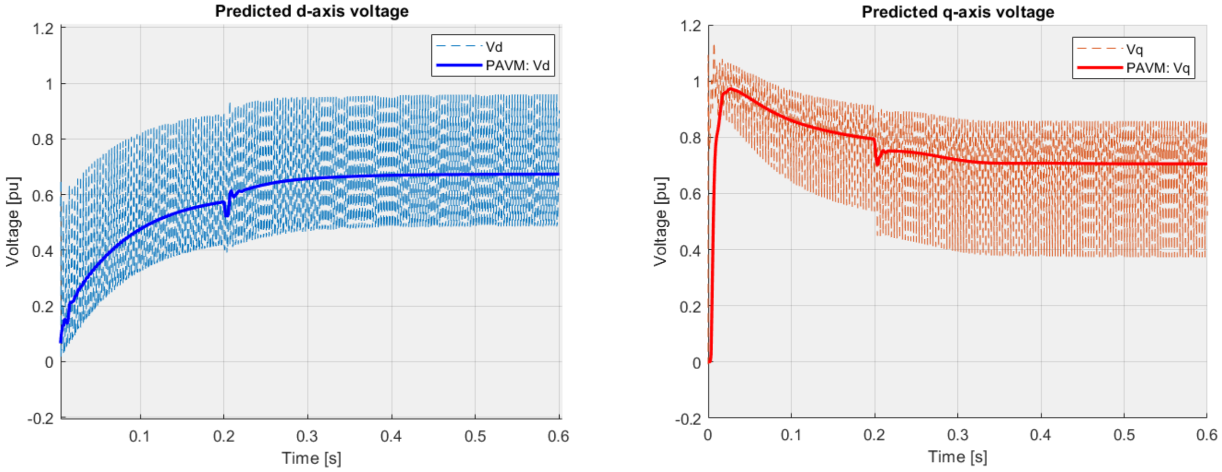

Figure 5. Predicted d- and q-axis voltage by the parametric average-value model (PAVM). Compared against the detailed model.

5. Analysis

Obtained results from steady-state simulations of the PAVM indicates good accuracy for both low and high dc-link loads. It is observed that the error of the model increases for higher loads, that might stem from the non-linear relationships at higher loading. Therefore, an improvement to this model could be to increase the number of support points particularly at higher loads, capturing the non-linear behaviour of the model better. This means doing more detailed simulations at higher loads on the dc-link. The PAVM is noticeable more accurate than the analytical model, except for predicting the dc-link voltage during load steps. This is believed to stem from the implementation of the PAVM in Simulink, where a too low dynamic impedance Z is calculated in the first time-step. The improvement of the accuracy of the PAVM for this variable is something that could be improved in the successive master thesis. However, it is only for the initial transient of the load step that the dc-link voltage is far off, as the steady-state behaviour is close to the detailed model. For the other variables: dc-link current and qd-axis voltage of the machine, the predicted values are very close to the actual values from the detailed model.

6. Conclusion

From the results it is shown that by modelling the diode rectifier with three algebraic functions implemented by look-up tables, both steady-state and transient behaviour of the machine-rectifier system can be captured with reasonable accuracy. In the author's master thesis, the work presented in this paper will be expanded to a more complex system consisting of a synchronous generator interfaced by a full-rated back-to-back voltage source converter, including representations of the turbine/governor system and excitation system. The reduced-order average-value model will be based on a real-life laboratory setup at NTNU, and if time allows, the accuracy of the full-scale average-value model will be assessed against this setup during laboratory testing

Appendices

A.1 Simulink System Parameters

Table 1. Synchronous Machine Parameters.

Parameter	Value
Sn	250 kVA
Vn	400 V
In	360 A
fn	50 Hz
Rs	0.0259 pu
Xl	0.09 pu
Xd	2.84 pu
Xd1	0.18 pu
Xd2	0.13 pu
Xq	2.44 pu
Xq2	0.36 pu
Td	0.08 s
Td2	0.019 s
Tq2	0.019 s

Table 2. Diode Rectifier and DC-link Parameters.

Parameter	Value
Rsnub	1000 Ohm
Csnub	1 uF
Ron	1 mOhm
Cdc	2 mF
Rload	0.9 Ohm →

B.1 abc-to-qd Transformation and Rectifier Reference System

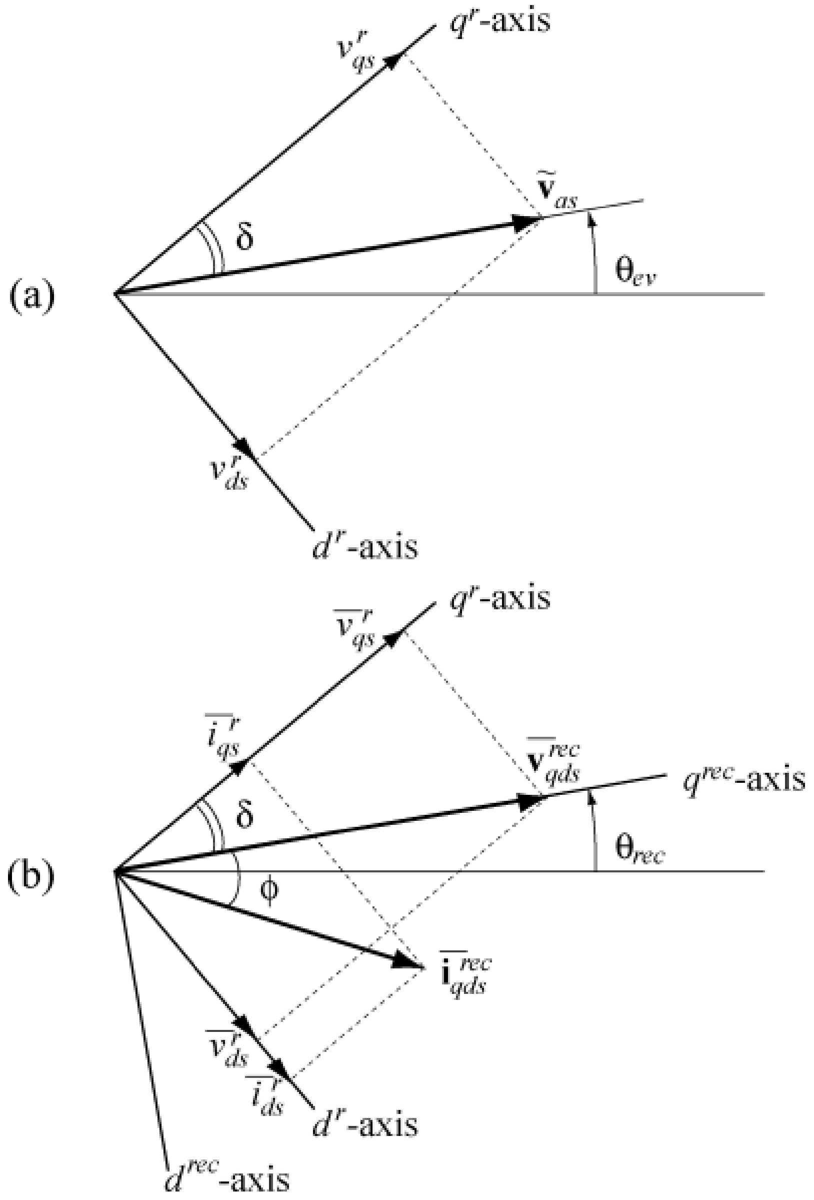

Figure B.1 The relationship between abc- and qd-variables (a), and the relationship between generator and rectifier variables (b). Source: Jatskevich et al. [**1**].

B.2 Support Points for the Parametric Functions

Table 3. Support points for look-up tables for the implementation of the diode rectifier average-value model.

Z	**α**	**β**	***ϕ***
0.9255	0.7813	1.322	0.2114
1.247	0.7816	1.312	0.2421
1.379	0.7811	1.309	0.2514
1.567	0.7801	1.306	0.2481
2.080	0.7777	1.298	0.2608
2.588	0.7747	1.295	0.2559
3.350	0.7709	1.290	0.2534
4.495	0.7662	1.284	0.2433
6.391	0.7603	1.280	0.2253
10.36	0.7544	1.273	0.2156
18.99	0.7479	1.266	0.2098
37.56	0.7433	1.253	0.2092
62.10	0.7373	1.242	0.2025
98.87	0.7313	1.236	0.2000

References

Examples taken from published papers:

[1] P. K. Steimer, O. Senturk, S. Aubert and S. Linder, "Converter-fed synchronous machine for pumped hydro storage plants," *2014 IEEE Energy Conversion Congress and Exposition (ECCE)*, Pittsburgh, PA, 2014, pp.4561-4567. doi: 10.1109/ECCE.2014.6954025

[2] M. Valavi and A. Nysveen, "Variable-Speed Operation of Hydropower Plants: A Look at the Past, Present, and Future," in *IEEE Industry Applications Magazine*, vol. 24, no. 5, pp. 18-27, Sept.-Oct 2018. doi: 10.1109/MIAS.2017.2740467

[3] J. Jatskevich, S. D. Pekarek and A. Davoudi, "Parametric average-value model of synchronous machine-rectifier systems," in *IEEE Transactions on Energy Conversion*, vol. 21, no. 1, pp. 9-18, March 2006. doi: 10.1109/TEC.2005.847974

[4] D. Borkowski, "Analytical Model of Small Hydropower Plant Working at Variable Speed," in *IEEE Transactions on Energy Conversion*, vol. 33, no. 4, pp. 1886-1894, Dec. 2018. doi: 10.1109/TEC.2018.2849573

[5] Chiniforoosh, Sina & Jatskevich, Juri & Atighechi, Hamid & Martinez-Velasco, Juan. (2014). Dynamic Average Modelling of Rectifier Loads and AC-DC Converters for Power System Applications. 10.1002/9781118694190.ch10.